MATERION

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TECHNICALTIDBITS

MATERION PERFORMANCE ALLOYS

THE GECKO AND THE DRAGON:
NONLINEAR SCALING EFFECTS

Scales can be bad news nowhere near the holidays. – How changes in scale affect the way that materials and environmental forces interact.

- **Scaling Laws**
- **Isometric Scaling**
- **Allometric Scaling**

Isaac Newton's laws described the physical workings of the universe so well that it took around 300 years of scientific and technological advancement to find exceptions.

Albert Einstein determined that these laws did not hold for objects at great distances or traveling near the speed of light. Niels Bohr, Max Planck and other pioneering quantum physicists figured out that the laws do not apply at the subatomic level. So, we know that the physics of problems change at the extremely small, extremely large, and extremely fast scales. Even in the everyday, intermediate-sized world we live in (where Newtonian physics do apply) there are some surprising effects due to changes in size, speed, strain rate, etc. These effects are covered by what is known as **scaling laws.**

For example, despite being very similar in shape, a gecko can easily climb up a wall but a Komodo dragon cannot. The reason is that the gecko is small enough (low mass) that minuscule Van der Walls attractive forces between the atoms in the wall and the atoms in the hairs on the gecko's feet are strong enough to overcome the gravitational pull of the earth. While the larger lizard would still experience the same Van der Walls forces, these forces are (thankfully) nowhere near strong enough to overcome the animal's weight. Therefore, the larger, more dangerous reptiles are earthbound.

Figure 1. Gecko (Left) **and Komodo Dragon** (Right)**.** Despite having very similar body shapes, the smaller reptile can easily climb walls while the larger reptile cannot, simply because it has a much larger ratio of surface area to mass.

Any quantity proportional to length, surface area, or volume will scale the same way as that particular dimension. Perimeter and surface tension at liquid-solid interfaces scale with length. Pressure, wind resistance, lift and drag, electrical resistance, resistive heating, etc. all are proportional to surface area (or cross sectional area). Mass, weight, frictional forces, kinetic energy, momentum, etc., correspond to volume. **Isometric scaling** is the name for when the length, width, and thickness of an object change at the same rate.

If we look at cantilever beam theory mass and weight increase linearly with the product of the length, width, and thickness, while the maximum safe force increases as the square of the thickness, increases linearly with width, but decreases linearly with the length. So, in isometric scaling, the force increase with the cube of the change in length, but the maximum safe force only increases with the square of the length! This means that beams will have to be made proportionally thicker and wider as the size goes up in order to withstand their own weight. Buckling columns show similar behavior.

$$\text{Maximum Force Load on Rectangular Cantilever Beam:}\quad F_{yield} = \frac{w \cdot t^2}{6L}\sigma_{yield}$$

$$\text{Critical Buckling Load on Rectangular Column:}\quad F_{critical} = \frac{w \cdot t^3 \cdot \pi^2}{48 \cdot L^2}E$$

The next issue of Technical Tidbits will speed-related scaling effects.

THE GECKO AND THE DRAGON: **NONLINEAR SCALING EFFECTS** (CONTINUED)

This effect was first described by none other than Galileo Galilee. (He was the first to really study beam theory, although much of his early work was not technically correct. Trust me, it's worth looking up.) Galileo noticed that small birds have very long, slender, lightweight bones. He realized that in progressively larger animals, the bones become proportionally thicker, shorter and denser, so they can support the additional weight. Allometric scaling is the name for the process where different dimensions such as length, width, and thickness change differently in proportion to size.

Figure 2 shows how in isometric scaling, a change in length of only two orders of magnitude results in 8 orders of magnitude change in surface to volume ratio (or surface to mass ratio). These ratios govern heating and cooling rates, buoyancy, terminal velocity in freefall, and many other physical phenomena. This is why very small objects have very different physical behavior than very large objects of similar shape and function. Behavior of very small objects such as geckos' feet or MEMS devices can be greatly affected by forces (electrostatic, Van der Walls, etc.) that are insignificant for larger objects.

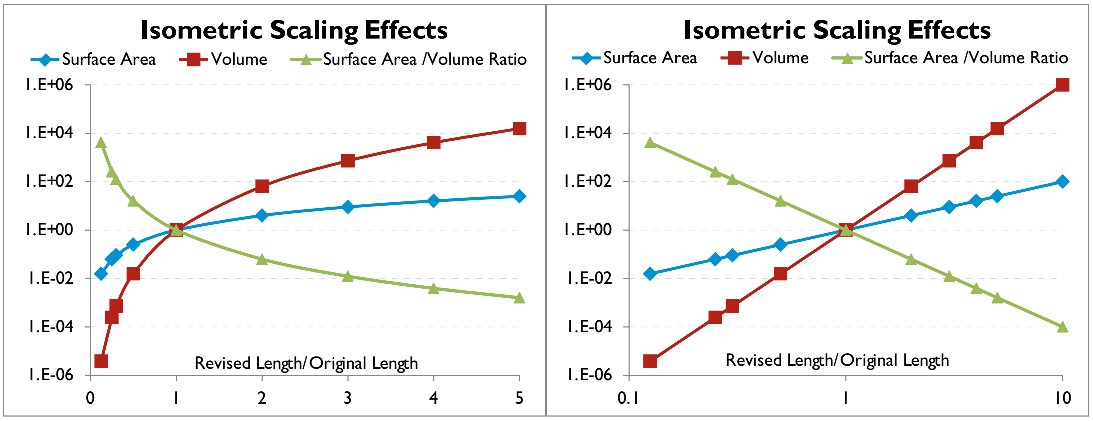

Figure 2. Isometric Scaling Relationships.
These charts show how surface area and volume change with length, assuming that the object scales uniformly in all 3 dimensions (isometric scaling). Note that as the length increases 100 times, the surface area increases 10,000 times, and the volume increases 10^{12} times. More importantly, the ratio of surface area to volume decreases 100 million times!

The bottom line is that a thorough knowledge of how all these physical quantities change with size is critical. If you take a design that is successful at one scale and simply make it larger or smaller, you may encounter unexpected failure modes, as the loading (forces, pressures, temperatures, etc.) will not necessarily scale at the same rates as the size. If you are making and testing scale models, you must be sure to use the proper scaling laws on all the loads so that the performance would truly have the same physics as the problem you are trying to model.

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