Issue No. 21 – September 2010 **Updated from Original March 2001 Publication**

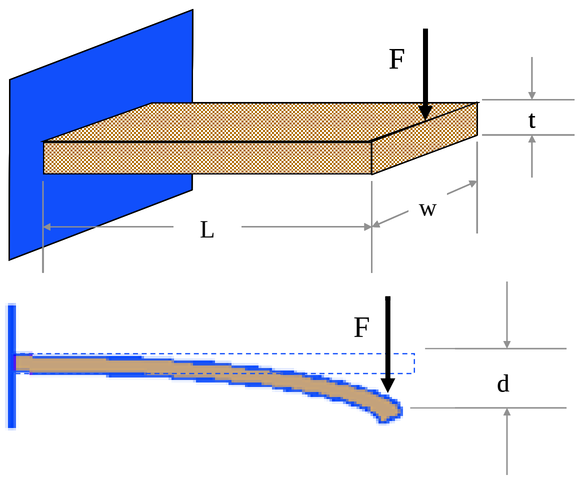
Cantilever Beams Part 2 - Analysis

Analyze This – A discussion of the ramifications of cantilever beam theory on spring design.

- **Allowable Deflection**
- **Contact Force at Yield**
- **Elastic Resilience**

The next issue of Technical Tidbits will discuss elastic resilience.

The last edition of Technical Tidbits introduced some key concepts of cantilever beam stress and force analysis. The equations for contact force and stress as a function of deflection are repeated in Figure 1. Both the stress and force depend on the elastic modulus of the beam material as well as the beam geometry. These are linear equations and hold true as long as the stress-strain relationship is linear. If the stress exceeds the elastic limit, and the material begins to yield, these relationships will no longer hold true.



Maximum Stress

$$\sigma_{\max} = \frac{3 \cdot E \cdot t}{2 \cdot L^2} \cdot d$$

Contact Force

$$F = \left[\frac{E \cdot w \cdot t^3}{4 \cdot L^3} \right] \cdot d$$

Figure 1. Straight cantilever beam with rectangular cross-section along with beam stress and force per unit deflection equations.

A key question to ask about any spring design is, "How much deflection can the spring tolerate before it yields ?" The stress equation can be rearranged to show deflection as a function of stress. The deflection at the yield (**allowable deflection**) can be found if the yield strength is inserted into the equation:

$$d_{yield} = \frac{2 \cdot L^2}{3 \cdot E \cdot t} \cdot \sigma_{yield}$$

The next obvious question to ask is, "How much contact force will the spring give me before it yields ?" The deflection at yield can be inserted into the force-deflection equation to come up with the **contact force at yield**:

$$F_{yield} = \left[\frac{E \cdot w \cdot t^3}{4 \cdot L^3} \right] \cdot d_{yield} = \left[\frac{E \cdot w \cdot t^3}{4 \cdot L^3} \right] \cdot \left[\frac{2 \cdot L^2}{3 \cdot E \cdot t} \right] \cdot \sigma_{yield} \qquad F_{yield} = \frac{w \cdot t^2}{6 \cdot L} \cdot \sigma_{yield}$$

Most springs are designed to generate as much contact force as possible, while withstanding a high range of deflections. Let us look at some methods of maximizing the deflection and force, referencing the four key equations:

Cantilever Beams Part 2 – Analysis (continued)

$$\sigma_{Max} = \frac{3 \cdot E \cdot t}{2 \cdot L^2} \cdot d \quad F = \left[\frac{E \cdot w \cdot t^3}{4 \cdot L^3} \right] \cdot d \quad d_{yield} = \frac{2 \cdot L^2}{3 \cdot E \cdot t} \cdot \sigma_{yield} \quad F_{yield} = \frac{w \cdot t^2}{6 \cdot L} \cdot \sigma_{yield}$$

Note that by increasing the width, we increase the contact force but do not affect the stress. If we increase the thickness of the material, we slightly increase the stress, and slightly decrease the allowable deflection. However, we get much larger gains in the force per unit deflection and the force at yield. Next, if we increase the length, we substantially reduce the force per unit deflection. However, we only slightly decrease the force at yield, because we reduce the stress per unit deflection and greatly increase the allowable deflection. If we increase the length and thickness by an identical percentage we get that same percentage decrease in stress per unit deflection, while not affecting the force per unit deflection. Additionally, the total allowable deflection and the total force at yield both increase by the same percentage. Therefore, we know that we can improve performance of the spring by increasing the length, width, and thickness of the beam.

The current trends in component design for computers, telecommunications, and automotive markets are towards miniaturization. This means that the length, width, and thickness of connectors and their contact springs must decrease in size. This will have a net negative impact on the contact force per unit deflection, maximum force, and allowable deflection. Therefore, choosing materials that have the material property benefits to overcome these geometrically imposed limitations becomes more important. For example, if we choose a material with a lower modulus, we get lower force and stress per unit deflection. The allowable deflection will increase, and the maximum stress will remain the same. So a low modulus material would seem to be a good option. However, there is very little variation in the elastic moduli of copper alloys, and the low modulus materials all have low yield strengths, which tends to offset the gains made by the lower modulus.

The obvious solution is to use a material with a high yield strength. As the yield strength increases, the allowable deflection and maximum force increase as well. This is the best of both worlds. Ultimately, the greatest advantage comes from a material with a high ratio of yield strength to elastic modulus. This important ratio has been given the name **"elastic resilience"**. The higher the resiliency, the better the contact will perform. This concept will be explored in a future edition of Technical Tidbits.

Written by Mike Gedeon of Brush Wellman's Alloy Customer Technical Service Department. Mr. Gedeon's primary focus is on electronic strip for the telecommunications and computer markets with emphasis on Finite Element Analysis (FEA) and material selection.

TECHNICAL TIDBITS

Brush Wellman Inc.
6070 Parkland Blvd.
Mayfield Heights, OH 44124
(216) 486-4200
(216) 383-4005 Fax
(800) 375-4205 Technical Service

BRUSH WELLMAN ALLOY PRODUCTS

Please contact your local sales representative for further information on stress and force analysis or other questions pertaining to Brush Wellman or our products.

Health and Safety

Handling copper beryllium in solid form poses no special health risk. Like many industrial materials, beryllium-containing materials may pose a health risk if recommended safe handling practices are not followed. Inhalation of airborne beryllium may cause a serious lung disorder in susceptible individuals. The Occupational Safety and Health Administration (OSHA) has set mandatory limits on occupational respiratory exposures. Read and follow the guidance in the Material Safety Data Sheet (MSDS) before working with this material. For additional information on safe handling practices or technical data on copper beryllium, contact Brush Wellman Inc.