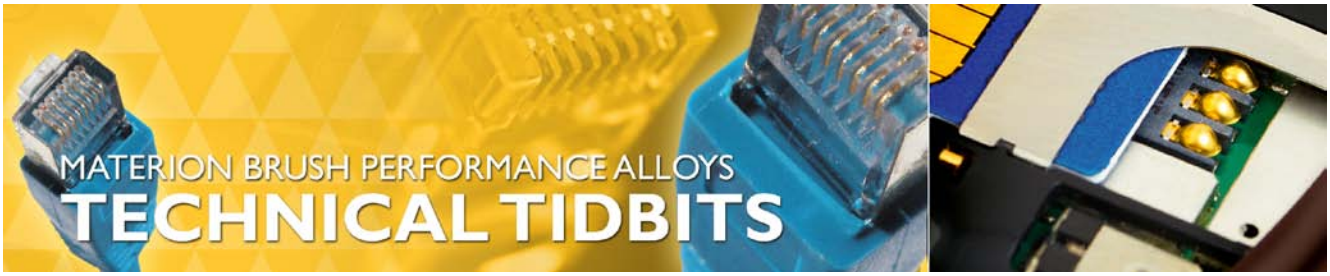

Issue No. 46 – October 2012 **Updated from Original May 2004 Publication**

Stress-Strain Concepts Revisited – Part 2

More stress in your life! – A continued discussion that goes back to the basics with an in-depth discussion of stress and strain concepts.

- **Tensile Strength**
- **Necking**
- **True Stress & Strain**
- **Toughness**
- **Elastic Resilience**
- **Modulus of Resilience**
- **Secant Modulus**
- **Tangent Modulus**

The next issue of Technical Tidbits will discuss yield strength and near-elastic properties of metals.

Last month's edition of Technical Tidbits started to explore the concepts of properties that can be obtained from a material's stress-strain curve. This is a continuation of that discussion.

After the yield strength is exceeded, the stress-strain curve continues to rise to a maximum point known as the **tensile strength** or the ultimate tensile strength. The strain up to this point is referred to as uniform strain, since the deformation in the specimen is uniform. However, once the tensile strength is exceeded, the test specimen begins to thin down, or **neck**, at some location along the length. All of the rest of the deformation to failure occurs at this point as the stress is concentrated in this area of reduced cross section. Since the engineering stress is determined by dividing the force load by the original cross sectional area, the engineering stress-strain curve begins to slope downward at this point, despite the fact that the actual stress in the specimen continues to increase. The curve continues until the specimen breaks when the fracture stress is reached.

True stress is determined by dividing the load by the smallest actual minimum cross sectional area of the specimen at any given time. The corresponding **true strain** at each point of the curve is computed by taking the natural logarithm of the actual length divided by the initial length. The true stress-strain curve increases after the tensile strength is reached. Figure 1 compares the engineering and true stress-strain curves for Brush Wellman Alloy 174 HT copper beryllium strip. Under uniform strain conditions (that is, up to the point of necking), the engineering and true stress and strain are related by the following equations:

$$\varepsilon = strain \qquad \sigma = stress$$

$$\varepsilon_{Engineering} = \frac{L_f - L_0}{L_0} \qquad \sigma_{Engineering} = \frac{F}{A_0}$$

$$\varepsilon_{True} = \ln\frac{L_f}{L_0} \qquad \sigma_{True} = \frac{F}{A}$$

$$\varepsilon_{True} = \ln\left(1 + \varepsilon_{Engineering}\right)$$

$$\sigma_{True} = \sigma_{Engineering}\left(1 + \varepsilon_{Engineering}\right)$$

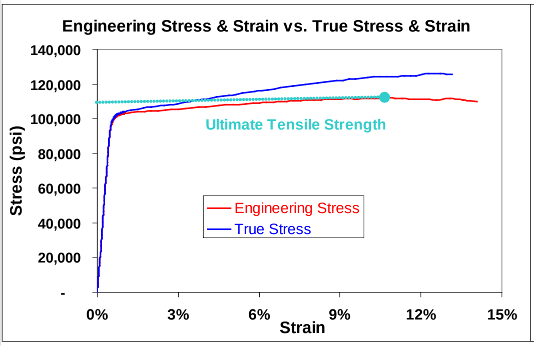

Figure 1. Alloy 17410 HT True and Engineering Stress-Strain Curves

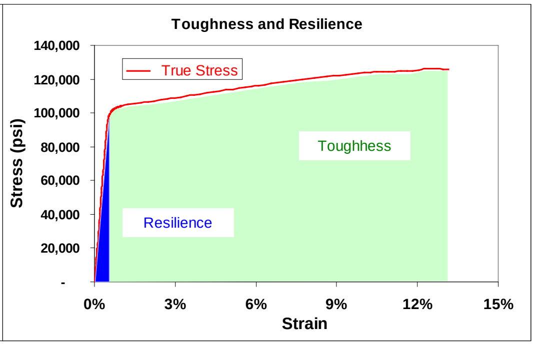

Figure 2. Toughness and Resilience

Stress-Strain Concepts Revisited (continued)

The amount of energy required to fracture a material is known as its toughness. The area underneath the stress-strain curve up to any given point is a measure of the energy density required to strain the material to that point. Therefore, the total area underneath the stress-strain curve provides a measure of the **toughness** of the material. (See Figure 2).

Elastic resilience is the amount of strain that a material can absorb without undergoing plastic deformation. It can be approximated by dividing the yield strength by the elastic modulus. Materials with high resilience can withstand greater amounts of deformation before yielding than materials with low resilience. The **modulus of resilience** is a measure of the strain energy density required to stress the material to the yield strength. It is the triangular area underneath the elastic portion of the stress-strain curve, as shown in Figure 2. It is approximated by taking the square of the yield strength and dividing by twice the elastic modulus.

The **secant modulus** at any point on the stress-strain curve is defined as the slope of the line from the origin to that point. Some materials do not show linear behavior in the elastic region of the stress-strain curve, so the secant modulus must be used in calculations instead of the elastic modulus. (Fortunately, the copper beryllium materials show well-defined elastic modulus values.) The **tangent modulus** at any point on the curve is defined by ASTM as the instantaneous slope of the curve at that point. However, in finite element analysis (FEA), the tangent modulus takes on a somewhat different meaning. It is defined as the slope of the true stress-strain curve from the yield strength to the ultimate tensile strength. This allows the entire stress-strain curve to be mathematically approximated by two straight lines: the elastic modulus and the tangent modulus. This bilinear approximation works very well for copper beryllium, since the elastic and plastic portions of their stress-strain curves are mostly linear. This can be seen in the example curve for Alloy 174 HT as shown in Figure 3.

Stress-strain curves generated by tensile testing provide a large amount of information on the expected performance of materials in engineered designs. Future editions of Technical Tidbits will focus on these properties in detail.

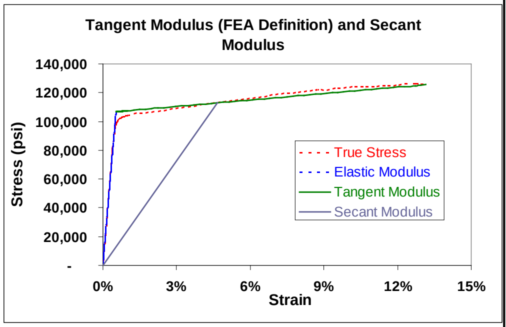

Figure 3. Tangent and Secant Moduli

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References:

Technical Tidbits Issue #27 "Tensile Testing"

Technical Tidbits Issue #1 "Why Good Designs Fail"

ASTM E8

ASTM E6

Please contact your local sales representative for further information on stress-strain curves or other questions pertaining to Materion or our products.

Health and Safety

Handling copper beryllium in solid form poses no special health risk. Like many industrial materials, beryllium-containing materials may pose a health risk if recommended safe handling practices are not followed. Inhalation of airborne beryllium may cause a serious lung disorder in susceptible individuals. The Occupational Safety and Health Administration (OSHA) has set mandatory limits on occupational respiratory exposures. Read and follow the guidance in the Material Safety Data Sheet (MSDS) before working with this material. For additional information on safe handling practices or technical data on copper beryllium, contact Materion Brush Performance Alloys