

Modulus operandi – An in-depth discussion on the elastic modulus and related properties.

- Elastic Modulus
- Tangent Modulus
- Secant Modulus
- Flexural Modulus
- Torsional Modulus
- Modulus of Rigidity (G)
- Bulk Modulus
- Volume Modulus of Elasticity

Elastic Modulus

(This issue of Technical Tidbits continues the materials science refresher series on basic concepts of material properties.) The elastic modulus is the material property that directly determines the spring rate of an electrical or electronic contact. It works with the geometry of a contact design to produce the normal force required for good electrical contact. Together with the yield strength, it determines how far a contact can be deflected, as well as how much springback there will be in a formed bend. It provides resistance to vibration and determines how much stress is necessary to cause buckling in forming processes. It is an important physical property for any given material, but it is often overlooked.

The **elastic modulus** can be denoted by a number of names: elastic modulus, Young’s modulus, modulus of elasticity, etc. It is often denoted by the letter E, with or without subscripts (E_x , E_y). Please note that there are other moduli that have completely different meanings: tangent modulus, secant modulus, flexural modulus, modulus of rigidity, etc. These will be discussed later.

Table 1 shows a number of design formulas that contain the elastic modulus. The first shows the reaction force as a function of deflection for a cantilever beam, with a high modulus providing high force. (The top formulas are for a generic beam of length L, those on the bottom are for a straight rectangular beam of thickness t and width w.) Recall that contact force is a critical parameter in electrical contact design, and must remain high enough to ensure good electrical contact. A high modulus allows for the easy generation of this force without the need for an overly large contact, and allows for miniaturization as well. The second formula shows stress as a function of deflection. This also increases with the modulus, since the stress in the contact is what generates the force. (This also illustrates why it is a good idea to use a material with a high yield strength as well as a high modulus.) The third formula shows the maximum deflection of a contact as a function of the material’s yield strength. Here, a high modulus would be a detriment, since it would limit the total amount of safe deflection. This effect can be countered with an increase in yield strength. (Astute readers will note that the ratio of elastic modulus to yield strength is also known as the elastic resilience of the material, and that one would like to maximize this quantity.) The last two formulas describe the critical force load and critical stress which would cause buckling in a vertically oriented cantilever beam. A high modulus is important here as well, if buckling during the forming process is a concern. (This most often comes into play during terminal crimping operations, since the barrel walls may buckle before they get a chance to fit snugly around the wire.)

For *linear elastic*, structural finite element analysis, the elastic modulus is the only material property used (other than Poisson’s ratio.) It is the only material property needed to determine stress distribution, reaction forces, displacements and safety factors. Of course, this assumes that the behavior of the part being modeled remains elastic. If the stress exceeds the yield strength of the material, more information is needed. *Nonlinear* structural analysis requires either the full stress-strain curve or a bilinear approximation, such as that shown in Figure 1. In nonlinear modeling, only two additional material properties are needed: the yield strength, and the **tangent modulus**. Here, the tangent modulus is defined as the slope of the stress-strain curve from the yield strength to the ultimate strength, on a *true* stress-strain curve. However, the ASTM definition of tangent modulus is significantly different from that used by FEA software. ASTM defines the tangent modulus at any point on the stress-strain curve as the slope of a line tangent to that point. This means that each point on the curve will have its own value of the tangent modulus, (except for points that are co-linear). On the other hand, FEA software recognizes only one value of the tangent modulus for every individual material.

The next issue of Technical Tidbits will discuss tensile strength.

Force	Stress	Maximum Deflection	Critical Buckling Load	Critical Buckling Stress
$F = \frac{3 \cdot E \cdot I}{L^3} d$ $= \frac{E \cdot w \cdot t^3}{4 \cdot L^3} d$	$\sigma = \frac{3 \cdot E \cdot c}{L^2} d$ $= \frac{3 \cdot E \cdot t}{2 \cdot L^2} d$	$d_{\max} = \frac{L^2}{3 \cdot c} \cdot \frac{\sigma_y}{E}$ $= \frac{2 \cdot L^2}{3 \cdot t} \cdot \frac{\sigma_y}{E}$	$P_{\text{crit}} = \frac{\pi^2 \cdot E \cdot I}{4 \cdot L^2}$ $= \frac{\pi^2 \cdot E \cdot w \cdot t^3}{48 \cdot L^2}$	$\sigma_{\text{crit}} = \frac{\pi^2 \cdot E \cdot I}{4 \cdot L^2 \cdot A}$ $= \frac{\pi^2 \cdot E \cdot t^2}{48 \cdot L^2}$

Table 1. Key design formulas that use the elastic modulus (assuming straight, rectangular cantilever beams).

Elastic Modulus (continued) The bilinear approximation works well for copper beryllium alloys such as the C17410 1/2 HT material shown in Figure 1. However, not all materials have such clearly linear regions. In some alloys, the elastic portion of the stress-strain curve may not be straight at all, but will be continuously curving. In this case, the **secant modulus** would be used as an approximation of the elastic modulus. The secant modulus is defined as the slope of a line from the origin of the curve to a specific point on the stress-strain curve. There will therefore be as many values for the secant modulus as there are points on the stress-strain curve. The secant modulus would have a maximum value at the origin and would gradually fall off as the stress and strain increase. (For alloys that do have a linear elastic modulus, the secant modulus would be equal to the elastic modulus and would remain the same value for the entire proportional region of the curve.) There will be more on the importance of this property in a future edition of Technical Tidbits.

The **flexural modulus** is the value of the elastic modulus as determined by a spring bend limit testing device. It is calculated using bending formulas from the force-deflection relationship. It is typically equal to the elastic modulus in tension. The **torsional modulus**, also known as the **modulus of rigidity** or **G**, is related to the elastic modulus. While the elastic modulus is the relationship between normal (axial) stress and strain, the torsional modulus is the relationship of shear stress and shear strain. It can be calculated from the elastic modulus by the following formula: $G = E / 2(1 + \nu)$, where ν is Poisson's ratio. The **bulk modulus**, also known as the **volume modulus of elasticity**, is the ratio of normal stress to uniform volumetric strain when a solid is compressed by hydrostatic pressure. It is calculated by the following formula: $B = \Delta p V / \Delta V$, where Δp is the change in pressure, and V and ΔV are the volume and change in volume, respectively. It also is found by the following: $B = E / 3(1 - 2\nu)$. As a matter of interest, the speed of sound in a material is equal to the square root of the ratio of its bulk modulus to its density.

The elastic modulus is an important property to consider when selecting a material, just as important as yield strength, conductivity or formability. Although often overlooked, the elastic modulus is a critical material property which should be factored into the material selection process. As a partial guide, Table 2 lists the elastic modulus for some strip materials commonly used in electronic connector applications. The copper beryllium alloys at the top of the list provide the largest elastic modulus, and thus improved reliability in connector designs.

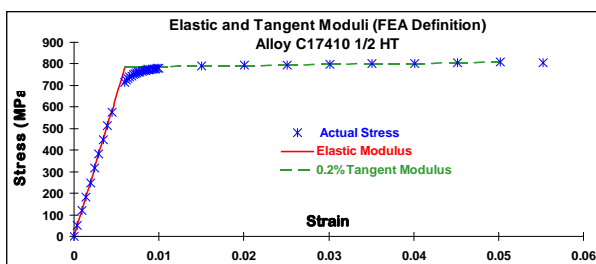


Figure 1. Elastic and Tangent Modulus as used in FEA.

Alloy	Mpsi	GPa	
C17410	CuBe	20	138
C17460	CuBe	20	138
C17200	CuBe	19	131
C19010	CuNiSi	19	131
C70250	CuNiSi	19	131
C70290	CuNiSn	18.5	128
C19900	CuTi	18	124
C42500	CuZnSn	17	117
C65400	CuSiSn	17	117
C68800	CuZnAl	17	117
C51000	CuSn	16	110
C52100	CuSn	16	110

Table 2. Elastic Modulus of common copper strip alloys.

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References:

Technical Tidbits Issue #27 "Tensile Testing"

ASTM E8

ASTM E6

Please contact your local sales representative for further information on the elastic modulus or other questions pertaining to Materion or our products.

Health and Safety

Handling copper beryllium in solid form poses no special health risk. Like many industrial materials, beryllium-containing materials may pose a health risk if recommended safe handling practices are not followed. Inhalation of airborne beryllium may cause a serious lung disorder in susceptible individuals. The Occupational Safety and Health Administration (OSHA) has set mandatory limits on occupational respiratory exposures. Read and follow the guidance in the Material Safety Data Sheet (MSDS) before working with this material. For additional information on safe handling practices or technical data on copper beryllium, contact Materion Brush Performance Alloys