Spring Types Part 5 – The Unique Stiffness Behavior of Belleville Washers

This series of six or so editions of Technical Tidbits will discuss various types of springs used in electrical contacts or sensors, and group them into six broad categories of similar function (cantilever beams, simply supported beams, torsion bars, Belleville washers, coil springs, and bellows & diaphragms). This month we will continue the discussion of Belleville washers.

The spring rate (stiffness) of a spring is the relationship between force and deflection. In many spring designs, this is a linear relationship over the majority of the deflection, resulting in a constant spring rate. However, the stiffness behavior of the Belleville washer is governed largely by the ratio of the free height to the washer thickness. By varying this ratio, a wide variety of force-deflection responses can be achieved, as shown in Figure 1 (and in many other sources). Because of the horrendous complexity of the equation governing the spring rate of the Belleville washer force, it is easiest to plot deflection as a percentage of deflection to flat, and spring force as a percentage of force at flat deflection.

Note that Belleville springs can be deflected beyond flat, if they are restrained around the edges and have enough room to invert themselves. Thus, the maximum deflection is approximately 200% of the deflection to flat. Note that the force-deflection curves for a given h/t ratio are symmetric about the flat deflection. Furthermore, note that since the spring rate is the slope of the force-deflection curve, there are certain configurations that can have a near zero or even a negative spring rate. That is, as the deflection increases, there is very little increase in force or even a decrease in reaction force, even if the material is nowhere near the yield strength. Note that the reaction force can even be negative, which happens when the spring snaps through and inverts itself. The magnitude of this negative force is basically the amount of force required in the opposite direction to snap the spring back to its original orientation.

Figure 1. Belleville Washer Stiffness. In most springs, the spring rate remains constant until either the material yields under stress or the spring deforms to the point where the geometry becomes different enough from the starting conditions that the equations no longer apply (such as when a straight cantilever beam becomes highly curved under load). At that point, the slope will decrease, but still remain positive. The spring rate of Belleville washers varies throughout the deflection, and may even turn negative under certain conditions.
Spring Belleville Washers (continued)

Let us examine each of the possibilities for spring rate. At a free height to thickness ratio of around 0.4 or less, the force-deflection curve is approximately linear, as seen in the curves for h/t ratios of 0.2 and 0.4. As the h/t ratio increases above 0.4, a “peak” starts to grow from the left side of the curve (before flat), and a “valley” starts to grow from the right side of the curve (after flat).

For an h/t ratio above 0.4 but less than about 1.41, the slope changes from a straight line to more closely resemble a cubic function. However, there is no local maxima or minima on these curves. For an h/t ratio of about 1.41, the slope becomes nearly horizontal around the deflection to flat. This means that the spring can be deflected further with almost no additional force, until well past flat. This is the beginning of snap-through type behavior.

<table>
<thead>
<tr>
<th>h/t ratio</th>
<th>Spring Rate Behavior</th>
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</thead>
<tbody>
<tr>
<td>≤ 0.4</td>
<td>Approximately linear</td>
</tr>
<tr>
<td>&gt; 0.4, &lt; 1.41</td>
<td>Resembles cubic equation. Slope starts greater than 1, then falls to ≈1 at flat, decreases to less than 1, then increases again.</td>
</tr>
<tr>
<td>1.41</td>
<td>Slope increases, becomes approximately 0 near deflection to flat, then increases again.</td>
</tr>
<tr>
<td>&gt; 1.41, &lt; 2.83</td>
<td>Increases to peak, quickly turns negative and passes through flat with a slope near -1, then reaches a minimum and increases again.</td>
</tr>
<tr>
<td>≥ 2.83</td>
<td>Increases to peak, turns negative, passes through flat with a slope ≈ -1, then reaches snap through point. Requires equal and opposite force to snap back to original shape.</td>
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</tbody>
</table>

Table 1. Spring Rate Behavior for Various h/t Ratios. Belleville washers have some unique stiffness characteristics that are found in no other spring configuration.

As the ratio increases above 1.41, definite peaks and valleys begin to form. The curve will reach a maximum point before flat, then the slope turns negative, generating a decreasing amount of contact force as the deflection increases until it reaches the minimum on the other side of flat, then the force required to further invert the washer begins to increase again.

When the h/t ratio exceeds 2.83, the snap-through behavior is in full effect, and the washer generates a negative contact force when deflected far enough. What this really means is that the washer will quickly snap through to its inverted state, and you cannot return the washer to its original state without applying an equal and opposite force to the other side.

These washers display a wide range of unique stiffness behaviors, making them an interesting choice of spring. No matter what kind of behavior you want, you are likely to find it in some configuration of Belleville Washer.

Written by Mike Gedeon of Materion Performance Alloys Marketing Department. Mr. Gedeon’s primary focus is on electronic strip for the automotive, telecom, and computer markets with emphasis on application development.

References:
- *Handbook of Spring Design* ©1993 Spring Manufacturers Institute
- Carlson, Harold *Spring Designer’s Handbook* ©1978 Marcel-Dekker, Inc.

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Materion Performance Alloys
6070 Parkland Blvd.
Mayfield Heights, OH 44124

Sales +1.216.383.6800 800.321.2076 BrushAlloys@Materion.com
Technical Service +1.216.692.3108 800.375.4205 BrushAlloys-Info@Materion.com

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