



## Statistical Analysis of S-N (Wöhler) Diagrams

(This issue of Technical Tidbits continues the materials science refresher series on basic concepts of material properties.) Last month's edition of Technical Tidbits discussed how a material's S-N (Wöhler) curve is derived. This week we will discuss how to obtain useful information from it.

The chart below shows an S-N (Wöhler) diagram for C17410 strip, in fully reversed bending. There is a great deal of scatter in the data, which is to be expected when a material is fully characterized. The red dashed line is simply a best fit trend line through these points. About 50% of the failures are above the trend line (representing longer life), and the other 50% are below the trend line, representing shorter life. So, the question to be asked is how such a chart can be used to statistically relevant predictive data.

### Reliable Reliability Information – A

discussion on how fatigue data can be statistically analyzed to provide more useful information to the design engineer.

- S-N Diagram (Wöhler Diagram)
- Weibull Distribution
- Normal Distribution

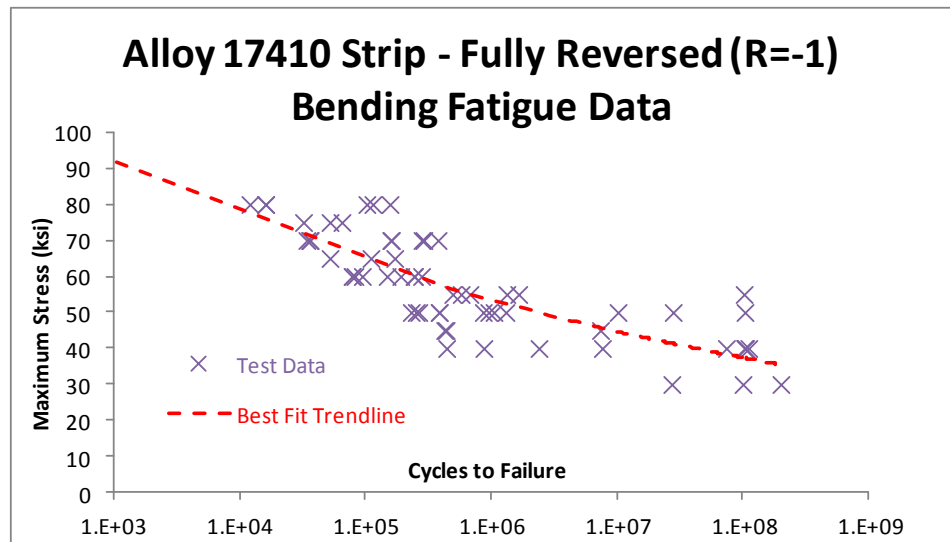


Figure 1. S-N Diagram with Best Fit Trend Line.

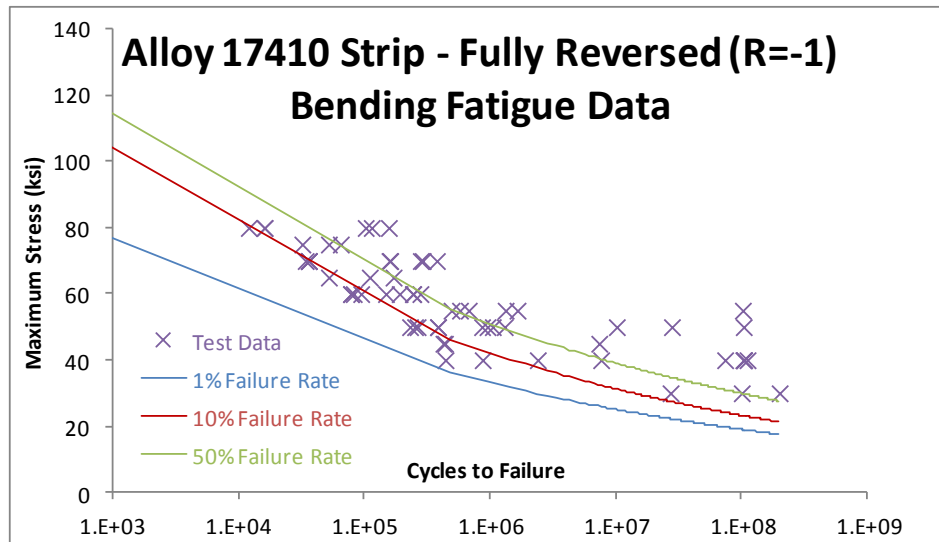
The large amount of scatter in the data makes the trend line nearly pointless, since about half of the failures occur before the predicted times, and about half would occur after.

The data plotted above are nice in that 5 samples were tested at each stress level, sufficient to statistically determine the expected distribution of the cycles to failure. The first step would be to analyze the data points at each applied stress level, to determine how they are distributed, such as normal (Gaussian), 1-parameter Weibull, 2-parameter Weibull, 3-parameter Weibull, etc. Once this information is determined, the resulting number of cycles that would correspond to a given failure rate could be determined. (Detailed step by step instructions for performing this kind of analysis are available from multiple sources, and there are multiple online tools and software packages that can perform the analysis for you.)

Once this analysis is performed, for each applied stress level one can generate a table of the number of cycles expected to correlate with a 1% failure rate, 5% failure rate, 10% failure rate, etc. These points can then be plotted on an S-N chart. Best fit trendlines can then be drawn through the points corresponding to a given failure rate, resulting in curves for 1% failure, 10% failure, 50% failure, etc. These statistically derived trendlines are much more useful than a best fit trendline through the raw data, since these can give you an idea of the expected failure rate over time at a given stress level, whereas a trendline through the raw data will only give you a rough idea of when half of your parts would fail.

The next issue of Technical Tidbits will continue the discussion on Fatigue.

**Statistical Analysis of S-N (Wöhler) Diagrams (continued)** Figure 2 below shows the data of Figure 1 replotted, this time with trend lines corresponding to various failure rates. (The individual data points for 1%, 5%, and 10% failure have been omitted for clarity. The accuracy of the curves increases with the number of samples at each given stress level. This determines the width of the failure band at each stress level. Furthermore, as the number of stress levels tested increases, the shape of the failure rate curves should improve in accuracy as well.



**Figure 2.** Improved S-N Diagram. At each stress level, a statistical analysis is performed to estimate the number of cycles at which 1%, 10%, and 50% of the samples are expected to fail. These statistically-derived points are then fit with trendlines to better define the failure envelope.

When setting the maximum design stress level in your component, it would be best to use such statistically analyzed curves. If you were to design to the best fit curve through the raw data points, on average, one half of the components would fail before reaching the desired lifetime. Using statistical analysis, coupled with appropriate nonlinear finite element stress and fatigue analysis, will ensure that the parts that you design will achieve the desired lifetime. Of course, there will always be statistical outliers and other failures due to special causes like manufacturing or material defects. Your goal as a designer is to prevent recurring, systemic failures due to simplified assumptions.

*Written by Mike Gedeon of Materion Brush Performance Alloys Customer Technical Service Department. Mr. Gedeon's primary focus is on electronic strip for the telecommunications and computer markets with emphasis on Finite Element Analysis (FEA) and material selection.*

## TECHNICAL TIDBITS

Materion Brush Performance Alloys  
 6070 Parkland Blvd.  
 Mayfield Heights, OH 44124  
 (216) 486-4200  
 (216) 383-4005 Fax  
 (800) 375-4205 Technical Service



**MATERION**

### References:

**ASTM E468:** Standard Practice for Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials

Bazovsky, Igor  
Reliability Theory and Practice  
 Prentice-Hall, Inc.,  
 Engelwood Cliffs, NJ  
 1961

Summerville, Nicholas  
Basic Reliability  
 Author House,  
 Bloomington, IN 2004

ASM Handbook Volume 19 Fatigue and Fracture  
 ASM International 1996

<http://www.weibull.com/ReliabilityEngineeringResources>

Please contact your local sales representative for further information on the fatigue or other questions pertaining to Materion or our products.

### Health and Safety

Handling copper beryllium in solid form poses no special health risk. Like many industrial materials, beryllium-containing materials may pose a health risk if recommended safe handling practices are not followed. Inhalation of airborne beryllium may cause a serious lung disorder in susceptible individuals. The Occupational Safety and Health Administration (OSHA) has set mandatory limits on occupational respiratory exposures. Read and follow the guidance in the Material Safety Data Sheet (MSDS) before working with this material. For additional information on safe handling practices or technical data on copper beryllium, contact Materion Brush Performance Alloys or your local representative.